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Microeconomic Models of Housing Markets

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Managing Editors

Prof. Dr. M. Beckmann
Brown University
Providence, RI 02912, USA

Prof. Dr. W. Krelle
Institut für Gesellschafts- und Wirtschaftswissenschaften
der Universität Bonn
Adenauerallee 24–42, D-5300 Bonn, FRG

Editor

Prof. Dr. Konrad Stahl
Lehrstuhl Wirtschaftstheorie, insbesondere Stadtökonomie
Universität Dortmund
Postfach 500500, D-4600 Dortmund 50, FRG

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THE DORTMUND HOUSING MARKET MODEL: A MONTE CARLO SIMULATION OF A REGIONAL HOUSING MARKET

Michael Wegener

Introduction

Work on the Dortmund housing market model began in 1977 at the Institute of Urban and Regional Planning of the University of Dortmund as part of a larger research project supported by the Deutsche Forschungsgemeinschaft. This ongoing project is directed towards the investigation of the long-term interactions between economic (sectoral, technological) change, locational choice, mobility, and land use in urban regions. For this purpose, a spatially multilevel dynamic simulation model of regional development was designed to simulate, within a concrete regional context,

- the location decisions of industry, residential developers, and households,
- the resulting migration and travel patterns,
- the land use development, and
- the impacts of public programs and policies in the fields of employment, housing, and infrastructure.

It was decided to use the urban region of Dortmund as a study region, including Dortmund and 18 neighboring communities with a total population of 2.4 million.

The intraregional component of this model system is the housing market model described here. The decision to model intraregional migrations as transactions in the regional housing market was based on the empirical evidence established by many surveys according to which household mobility within urban regions, unlike long-distance mobility, is almost exclusively determined by housing considerations, i.e. by changing housing needs during their life cycle.

Accordingly, the housing market model developed is primarily a model of choice behavior of households and landlords subject to economic and non-economic choice restrictions. On the demand side, considerable effort has devoted to modeling the life cycle of households and their concurrently changing decision situations and preferences. On the supply side, the housing stock is changed through aging, public housing programs, or private construction by housing investors or owner-occupants. The model differs from other housing market models by the stochastic technique by which it simulates the market clearing process and by the fact that it is incorporated into a larger model framework of regional development, industrial location, household mobility, and land use.

The following description of the model is done in four sections. Section 1 is a brief summary of the whole model system of which the housing market model is a part. In Section 2, the major hypotheses about the working of the housing market underlying the model are outlined. Section 3, the actual model description, contains detailed information about the model structure, equations, and computational techniques used. In Section 4, the data sources of the model and the techniques applied to calibrate its parameters are discussed.

1. The Model Environment

As mentioned above, the housing market model discussed here is part of a larger simulation model of regional development, industrial location, household mobility, and land use. The whole model is organized in three spatial levels:

(1) a macroanalytical model of the economic and demographic development of 34 labor market regions in North-Rhine Westphalia,

(2) a mesoanalytic model of intraregional location and migration decisions in 30 zones of the urban region of Dortmund,

(3) a microanalytic model of land use development in one or more urban districts of Dortmund.

The North-Rhine Westphalia model constitutes the first level of the three-level model hierarchy. Its purpose is to forecast the labor demand by industry in the 34 labor market regions of the state of North-Rhine Westphalia and the migration flows between them, subject to exogenous employment and population projections for the whole state (Schönebeck, 1983).

The results of the North-Rhine Westphalia model serve as the framework for simulating the intraregional location and migration decisions of industry, residential developers, and households at the second level of the model hierarchy within the Dortmund region model. Its study area is the urban region of Dortmund defined as Dortmund's commuter catchment area. It includes the Dortmund labor market region of the North-Rhine Westphalia model and a ring of communities in adjacent labor market regions. The 30 geographical subdivisions of the Dortmund region are called zones.

On the third level of the model hierarchy, the land use development allocated to zones on the second level is further distributed to individual tracts within one or more zones of Dortmund. Figure 1 shows the study areas of the three model levels and their relationship to each other.

The information flows between the model levels are established through the recursive temporal structure of the model system. On all three levels, the model proceeds in discrete time intervals or periods from a base year to a simulation horizon. Typically, the duration of a period is two years. Up to ten periods, or 20 years, can be simulated in one run. Like in all recursive models, the end state of one period equals the initial state of the next one. At each break point between periods, information concerning the next period is transmitted from one model level to the next lower one. Presently, only top-down information flows have been implemented, although also bottom-up feedback is envisaged for future work.

The housing market model is located on the second, or urban region, level. This model level represents in effect a comprehensive model of spatial urban development encompassing model sectors of employment, population, residential and nonresidential buildings, public facilities, and transport. Its aggregation level is neither macro, nor is it really micro, but may be characterized as mesoanalytic:

- Throughout it uses aggregate data, but in a relatively fine stratification. For instance, employment is classified by 40 industrial sectors and 4 skill levels, population by nationality, sex, and 20 age groups, households by 120 (30) household types, housing by 120 (30) dwelling types, land use by 30 land use categories, etc.
- Its spatial subdivisions (zones) range in population between 40,000 and 60,000 in the center of the region, but include also at its periphery considerably larger industrial centers such as Bochum (population 400,000) and Hagen (230,000).

In the Dortmund region model, the housing market is only one of several separate, but closely interlinked spatial markets: the transport market, the regional labor market, the housing market, the construction market, and the market for industrial and commercial buildings. On these markets, private (individual or corporate) actors such as travelers, workers, households, landlords, developers, and entrepreneurs interact through competitive choice processes. Choice in the markets is constrained by supply (transport supply, vacant jobs, vacant housing, vacant land, vacant industrial or commercial floorspace) subject to public policies and

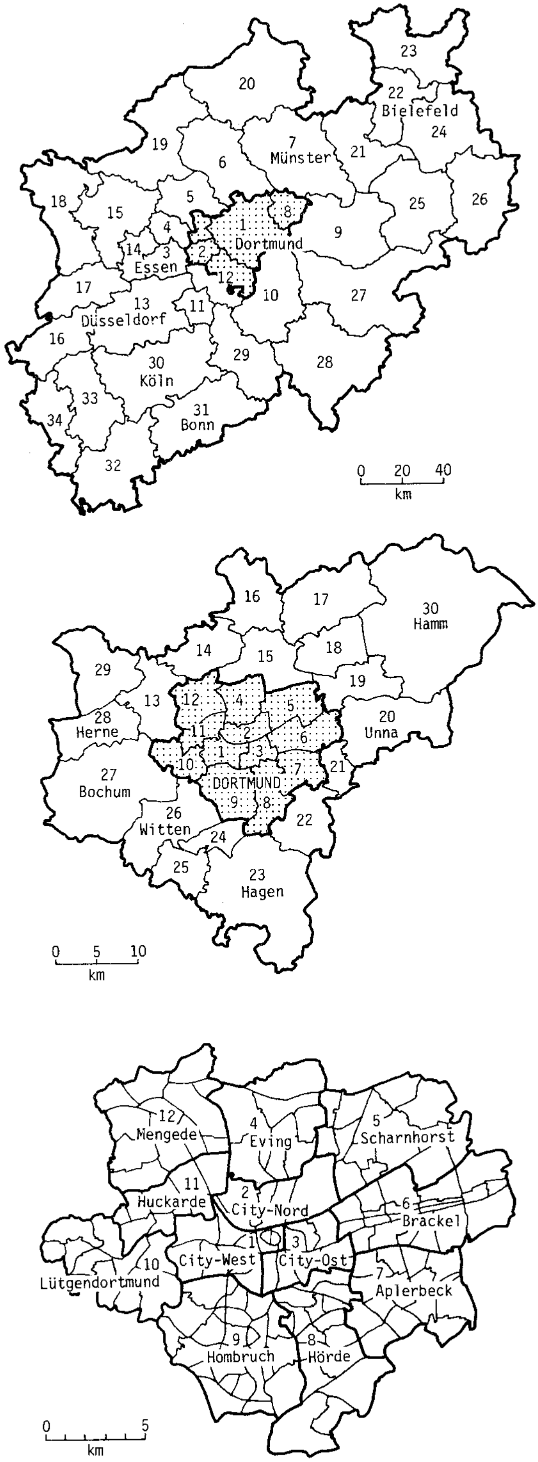


Figure 1. The three model levels.

regulations, and is guided by utility or attractiveness and preferences. Utility or attractiveness is generally an actor-specific aggregate of within-place attractiveness and between-places accessibility, and price.

The Dortmund region model simulates the aggregate behavior of these market actors and its spatial consequences for the urban region subject to three kinds of exogenous inputs:

a) regional forecasts of employment by sector for the whole region and of inmigration into and outmigration out of the region;
b) demographic, monetary, and technological parameters specifying long-term socioeconomic and technological trends originating outside of the region;
c) spatially and temporally disaggregated policies in the fields of landuse planning (zoning), housing construction, industrial development, public infrastructure, and transport.

Conditional on these exogenous inputs, the model endogenously predicts for each simulation period:

- the travel pattern,
- the aging of population, households, jobs, and buildings.
- the relocation and new construction of workplaces,
- the demolition, rehabilitation, and new construction of housing,
- the intraregional migration.

Except in the transport submodel, no equilibrium assumptions are made. In fact, the model never arrives at a general equilibrium within a simulation period. This does not imply that the model lacks negative feedback mechanisms working within or between the spatial markets. However, these feedbacks are always lagged and come into effect only in later simulation periods.

2. Model Hypotheses

The model is eclectic with respect to theory. Its main theoretical foundation is utility maximization, but this principle is enriched and made more realistic by a variety of assumptions about behavior subject to incomplete information and uncertainty, such as elimination by aspects, satisficing, adaptation, and learning.

As the housing market is modeled in the context of urban development at large, the hypotheses underlying the model design are embedded into a set of assumptions about the urban development process. The model sets out from the observation that in the recent past the development of large urban areas in highly industrialized countries has been characterized by a deglomeration or suburbanization process resulting in high growth rates at the periphery of urban regions at the expense of the city centers. The main causes of suburbanization have been:

- high demand for floorspace in the city centers for retail and offices, resulting in rising land prices and rents;
- decreasing attractiveness of living in the city centers because of traffic congestion, noise, air pollution, unavailability of parking space, lack of recreation facilities like parks, playgrounds, etc.;
- changing living and consumption patterns caused by rising incomes and reductions in daily/weekly/yearly work time leading to
  - smaller households, less children,
  - higher demand for housing space per capita,
  - more leisure time for recreation, sports, outdoor living,
  - higher emphasis on housing quality and location,
  - growing awareness of environmental qualities such as quietude, clean air, nature;
- improved accessibility of peripheral locations through highway construction, new public transport lines, and higher car availability;
- government support of home ownership through public subsidies and tax benefits;
- a public finance system forcing communities to compete for jobs and population.

The consequences of the exodus of people and jobs from the urban core have been monofunctionality of the city centers, increased spatial segregation of age and income groups, high expenses for public facilities and transportation, and urban sprawl at the periphery. All this, together with the loss of tax income, have made suburbanization a serious problem for many cities.

The problem is most severely felt where the region at large fails to compensate for local losses of employment and population. This is the case in most large cities of the Ruhr region like Dortmund which, due to the decline of the coal mining and steel industry, have experienced continuous losses of employment and population during the last fifteen years, while most of the growth of the region has been attracted by the metropolitan centers in the Rhine valley, namely Düsseldorf and Cologne.

The situation is aggravated by more recent trends of overall economic recession, high energy prices, growing unemployment and, for the first time since the post-war period, shrinking real incomes. Moreover, technological revolutions like the diffusion of microprocessors and new telecommunications threaten to dramatically change traditional patterns of activities, mobility, and location that seemed to be reliable and stable in the past.

All these trends and tendencies, taken together, make the future course of urbanization an extremely uncertain issue. Will suburbanization persist in a region with overall decline of employment and population? Will a declining population continue to demand ever more housing space as it did in the past? Will rising energy costs and transport prices enforce a more condensed or a more dispersed pattern of employment and residential location? Will telecommunication, office work at home, remote shopping or banking etc. substantially reduce the need for intraurban travel and gradually dissolve the spatial linkage between workplaces and residences? Will a decline in real income due to increased part-time employment affect the volume, composition, and spatial distribution of housing demand and eventually housing supply?

Many, if not most of these questions relate to the housing market, and this is why it has a central position in the Dortmund region model. The housing sector establishes the link between population and physical structure. It is here where long-term demographic and social developments such as changes in fertility, household formation patterns, income distribution, life styles, and consumption patterns have their impact on the physical structure of the region in the form of changing demand for housing. On the other hand, the existing housing stock constitutes the supply side of the housing market and thus in the short run determines the spatial distribution of population and all migration. Finally, new housing construction largely determines the future directions of spatial growth in the region.

More technically, the housing market is the place where households trying to satisfy their housing needs interact with landlords trying to make a profit from earlier housing investments. Housing investment or housing production decisions are not part of the housing market in this narrower sense, but are effected on the construction market, which is separate, but closely related to it.

The principal actors in the housing market (in the narrower sense) are the households exercising housing demand and the landlords offering housing supply. The modeling of their behavior proceeds from the following hypotheses:

- Household mobility within urban regions is largely determined by housing considerations.
- The housing demand of a household depends mainly on its position in its life cycle and its income.
- The satisfaction of a household with its housing situation can be represented by a utility function with the dimensions dwelling unit size and quality, neighborhood quality, location in the region, and housing cost.
- The willingness of a household to move is related to its dissatisfaction with its housing situation. A household willing to move does move if it finds a dwelling that gives it significantly more satisfaction than its present one.
- After a number of unsuccessful attempts to find a dwelling, a household adjusts its demand or abandons the idea of a move.
- Households have only limited information about the supply on the housing market; this limitation is related to their education and income.
- There are on the housing market local as well as social submarkets which are separated by economic and noneconomic barriers.
- Supply on the housing market is highly inelastic in the short run. There is practically no price adjustment in short market periods; quality or quantity adjustment is delayed by long construction times.

Quality or quantity adjustment of housing supply, i.e. maintenance/upgrading or new construction, occur on the construction market, where they have to compete with other land or building uses. The construction market is the place where owners of buildings or vacant land interact with investors who want to invest into buildings for sale, rent, or for their own use. On the construction market, different land and building uses compete with each other subject to restrictions specified in the land use or zoning plan.

The principal actors on the construction market with respect to housing supply are housing investors and land owners. Their behavior is modeled according to the following hypotheses:

- Quality adjustment of housing supply occurs through maintenance and/or upgrading investments. These investments are a function of the expected rent increase in that submarket.
- The attractiveness of a location (site) for a housing investor can be represented by a utility function with the dimensions site suitability, neighborhood quality, location in the region, and price.
- The supply of land suited for residential use is specified in the land use or zoning plan.
- If land supply exceeds demand, more attractive sites are likely to be developed sooner than less attractive ones.
- Where land demand exceeds supply, land owners can under certain restrictions provide additional land by demolition of existing buildings with less profitable types of use.
- If different types of land or building use compete for a particular piece of land, the land owner will normally sell the land to the most profitable type of use.

It will now be shown how these model hypotheses are reflected in the actual model implementation.

3. Model Structure

Housing demand and housing supply are represented in the model by households and dwellings classified by type. Households in each zone are represented as a four-dimensional distribution of households cross-classified by

- nationality (native, foreign),
- age of head (16-29, 30-59, 60+ years),
- income (low, medium, high, very high),
- size (1, 2, 3, 4, 5+ persons).

Similarly, housing of each zone is represented as a four-dimensional distribution of dwellings cross-classified by

- type of building (single-family, multi-family),
- tenure (owner-occupied, rented, public),
- quality (very low, low, medium, high),
- size (1, 2, 3, 4, 5+ rooms).

All changes of households and housing during the simulation are computed for these 120 household types and 120 housing types. However, where households and housing are cross-classified together, these household types and housing types are aggregated to H household and K housing types, with H and K not exceeding 30. Table 1a shows the 30-type household classification, Table 1b the 30-type dwelling classification presently used.

No.	Nationality	Age of head	Income group	Persons
1	native	16-29	low	1
2	native	16-29	low	2
3	native	16-29	low	3
4	native	16-29	low	4+
5	native	30-59	low	1
6	native	30-59	low	2
7	native	30-59	low	3
8	native	30-59	low	4+
9	native	16-59	medium	1
10	native	16-59	medium	2
11	native	16-59	medium	3
12	native	16-59	medium	4+
13	native	16-59	high	1
14	native	16-59	high	2
15	native	16-59	high	3
16	native	16-59	high	4+
17	native	16-59	very high	1
18	native	16-59	very high	2
19	native	16-59	very high	3
20	native	16-59	very high	4+
21	native	60+	low	1
22	native	60+	low	2
23	native	60+	low	3+
24	native	60+	medium	1
25	native	60+	medium	2
26	native	60+	medium	3+
27	native	60+	high/very high	1-2
28	native	60+	high/very high	3+
29	foreign	all	all	1-2
30	foreign	all	all	3+

Table 1a. 30-type household classification.

No.	Type of building	Tenure	Quality group	Rooms
1	single-family	all	all	1-3
2			to medium	4+
3			high	
4	multi-family	owner-occupied	to medium	1-3
5				4+
6			high	1-3
7				4+
8		rented	very low	1-2
9				3
10				4
11				5+
12			low	1-2
13				3
14				4
15				5+
16			medium	1-2
17				3
18				4
19				5+
20			high	1
21				2
22				3
23				4
24				5+
25		public	very low/ low	1-2
26				3
27				4+
28			medium/ high	1-2
29				3
30				4+

Table 1b. 30-type dwelling classification.

The cross-classification of households and housing is performed in the <u>occupancy matrix</u>. The occupancy matrix $\underline{R}$ of a zone is an H x K matrix representing the association of households with dwellings in the zone. Each element R_{hk} of the matrix contains the number of households of type h, h = 1, ..., H , k = 1, ..., K , the total matrix contains all households occupying a dwelling or all dwellings occupied by a household.

In addition, there are for each zone three vectors representing households without a dwelling or dwellings without a household. $\underline{H}^s$ is a H x 1 vector of subtenant households, and $\underline{D}^v$ is a 1 x K vector containing dwellings newly constructed in the previous period and released to the market now.

By incorporating the zonal dimension i, $i = 1, \ldots, I$, the matrix $\underline{R}$ becomes three-dimensional, and the vectors $\underline{H}^s$, $\underline{D}^v$, and $\underline{D}^n$ become two-dimensional matrices. $\underline{R}$, $\underline{H}^s$, $\underline{D}^v$, and $\underline{D}^n$ are a complete representation of the household/housing system of the model at the outset of the simulation period. All changes occurring to households and housing during the period can be represented by transitions into, within, or out of these four matrices.

The number and variety of changes that can occur to households and housing during a simulation period is enormous. Households come into existence, grow, get older, separate or merge, get more or less income, finally shrink and disappear. Dwellings are built, maintained or upgraded, or deteriorate and eventually are torn down. The association of households with dwellings changes in a seemingly uninterrupted succession of occupations and vacations, and this leads to changes of the composition and price of housing supply. In reality, all these changes occur in a continuous stream of closely interrelated events. However, there exists presently no feasible model capable of capturing all these changes in one integrated approach. Hence different model types are used, each one focusing on a particular subset of changes. Of course this implies that feedbacks that exist between changes modeled in separate models may be ignored.

Two principal kinds of changes of households and housing can be distinguished: changes that are decision-based and changes that are not. This distinction is relevant for choosing an appropriate model. For instance, migration and housing investments are normally based on rational decisions and can be and should be modeled as such. The aging of households and dwellings, however, depends only on the course of time and can thus best be modeled by probabilistic transition rates. Other changes are in reality decision-based, such as changes of household status through births, marriage, or divorce, but cannot be modeled causally at the chosen level of aggregation, and are therefore usually also modeled probabilistically. Still other changes are merely consequences of events occurring in other sectors of the model, e.g. changes of household income due to employment changes in the economic submodel. Such changes are exogenous to the household model. A last category of changes consists of genuinely exogenous changes, i.e. changes directly specified by the user such as public housing programs.

Following the above classification, in the Dortmund model changes of households and housing are modeled in six different submodels in this sequence:

(1) Aging of households and housing, including other changes of demographic changes of household status, are modeled in the aging/filtering submodel.
(2) Changes of household income induced by the economic submodel are modeled in the income change submodel.
(3) Changes of the association of households with dwellings are modeled in the market clearing or migration submodel.
(4) Public housing programs specified by the model user are executed in the public housing construction submodel.
(5) Private housing maintenance/upgrading and new construction investments are modeled in the private housing construction submodel.
(6) Changes of housing and land prices are modeled in the price adjustment submodel.

3.1 Aging/Filtering

A first group of changes of households and housing includes all changes that in the model are treated as merely time-dependent. For households such changes include demographic changes of household status in the household's life cycle such as aging and death as well as birth, marriage, and divorce and all new or dissolved households resulting from these changes, plus change of nationality by naturalization. On the housing side, they include deterioration by aging (filtering down the quality scale) and eventually demolition. However, all economically induced changes of household income are left to the subsequent income change submodel. All changes of housing occupancy connected with migration are left to the market clearing submodel, i.e. the aging/filtering submodel ages all households and housing without moving them relative to each other.

This is accomplished by a semi-Markov model with dynamic transition rates. A transition rate is defined as the probability that a household or dwelling of a certain type changes to another type during the simulation period from time t to time t+1. The transition rates are computed as follows: The time-dependent changes to be simulated are interpreted as events occurring to a household or dwelling with a certain probability in time interval t, t+1. These basic event probabilities and their expected future development are determined exogenously or are taken from the demographic submodel. Eleven basic event probabilities have been identified for each of the three household age groups:

1 change of nationality,
2 aging,
3 marriage,
4 birth, native,
5 birth, foreign,
6 relative joins household,
7 death,
8 death of child,
9 marriage of child,
10 new household of child
11 divorce,

and two for the four housing quality groups:

1 deterioration,
2 demolition.

Not all household events occur to every household. Some are applicable only to singles, some only to families, some only to adults, some only to children. Some household events are followed by housing events, and vice versa: where a household dissolves, a dwelling is vacated, and where an occupied dwelling is demolished, a household is left without a dwelling. The housing events contain only those changes of the housing stock which can be expected to occur under normal conditions in any housing area, i.e. a normal rate of deterioration and demolition. More demolition may occur in the industrial location submodel (not discussed here), where housing may have to make way for industrial or commercial land uses. Maintenance/upgrading and new housing construction are assumed to be demand-generated, i.e. decision-based and are therefore treated in the private housing construction submodel.

The basic event probabilities are aggregated to transition rates between household or dwelling types in two matrices, $\underline{h}_i$ for households and $\underline{d}_i$ for dwellings, using the disaggregate (120-type) household and housing distributions of each zone as weights. The matrices $\underline{h}_i$ and $\underline{d}_i$ are of dimensions H x H and K x K, respectively, where the rows indicate the source state and the columns the target state. Most events are independent of each other and can be aggregated multiplicatively; but some exclude others, i.e. are the complement of each other. Multiplication of $\underline{h}_i$ and $\underline{d}_i$ with the occupancy matrix $\underline{R}_i$ yields the occupancy matrix aged by one simulation period:

$$\underline{R}_i(t+1) = \underline{h}_i'(t,t+1)\underline{R}_i(t)\underline{d}_i(t,t+1) \tag{1}$$

where t indicates the beginning and t+1 the end of the current simulation period, and $\underline{h}_i'$ is the transpose of $\underline{h}_i$. This procedure assumes that all households of type h in zone i have the same transition rates, no matter in which dwelling they live, and vice versa.

Special provisions are necessary for events which create new households without a dwelling or new vacant dwellings. New households without a dwelling $\underline{H}_i^n$ may be generated by marriage of child, new household of child, or divorce:

$$\underline{H}_i^n(t,t+1) = \underline{h}_i^n(t,t+1)[\sum_k \underline{R}_{ki}(t) + \underline{H}_i^s(t)] \tag{2}$$

where $\underline{h}_i^n(t,t+1)$ is an H x H matrix containing current household formation probabilities and $\underline{R}_i(t)$ and $\underline{H}_i^s(t)$ are households occupying a dwelling and subtenant households as defined above. An element $h_{hh'}^n(t,t+1)$ of this matrix is defined as the probability that a new household of type h is produced by a household of type h' in zone i during the simulation period. Another way that a household without a dwelling may be generated is by demolition of a dwelling. Demolitions $\underline{H}_i^d$ are calculated as

$$\underline{H}_i^d(t,t+1) = \underline{R}_i(t)\underline{d}_i^d(t,t+1) \tag{3}$$

where $\underline{d}_i^d(t,t+1)$ is a K x 1 vector of demolition rates of housing types. Similarly, new vacant dwellings $\underline{D}_i^d$ may be generated by dissolution of households

$$\underline{D}_i^d(t,t+1) = \underline{h}_i^d(t,t+1)\underline{R}_i(t) \tag{4}$$

where $\underline{h}_i^d(t,t+1)$ is a 1 x H vector of dissolution rates of households aggregated from basic events like marriage, relative joining household, and death. Of course, new vacant dwellings may also result from housing construction, but this is effected in the public housing and private housing construction submodels.

In addition, it is necessary to age households and dwellings outside of the matrix $\underline{R}$, as also households without dwellings get older, and vacant dwellings deteriorate or may be torn down:

$$\underline{H}_i(t+1) = \underline{h}_i'(t,t+1)[\underline{H}_i^s(t) + \underline{H}_i^d(t,t+1) + \underline{H}_i^n(t,t+1)] \quad (5)$$

$$\underline{D}_i(t+1) = [\underline{D}_i^v(t) + \underline{D}_i^d(t,t+1) + \underline{D}_i^n(t-1,t)]\underline{d}_i(t,t+1) \quad (6)$$

where $\underline{D}_i^n(t-1,t)$ is the 1 x K vector of dwellings newly constructed in zone i in the preceding period. In equations (5) and (6) all households without a dwelling and all vacant dwellings of a zone are consolidated into the two matrices $\underline{H}$ and $\underline{D}$ for use in the market clearing model.

3.2 Income Change

The four household income groups used in the model are defined in terms of BAT (Federal Employment Salary Regulations) levels as follows:

1 Households having no or a very low earned income below the BAT; households which live on welfare or are supported by relatives; students, apprentices. In 1970, these households represented about 3.6 percent of all households.

2 Households having a low to medium income (equivalent to BAT VI and less). These households consist of blue collar and clerical white collar workers and represent about 82.7 percent of all households.

3 Households having a medium to high income (equivalent to BAT III-V). These households consist of medium grade white collar workers and public servants and represent about 10.1 percent of all households.

4 Households having a high to very high income (equivalent to BAT II and higher). These households earn their income by managerial and professional work and represent about 3.8 percent of all households.

At the beginning of the simulation period, disposable incomes and housing, shopping, and transport budgets of these household income groups are updated according to exogenously specified projections. Housing budgets include housing allowances and other public subsidies and are therefore different for owner-occupiers and renters.

During the simulation period, changes of the income distribution of households may be induced by changes of employment in the economic sector of the model. It is assumed that unemployment means that a household drops from one income group to the next lower one. Conversely, new employment means that a household is promoted by one income group.

Changes of employment are generated in the employment submodel (not discussed here) as job losses $E^r_{sj}(t,t+1)$ and new jobs $E^n_{sj}(t,t+1)$ of sector s a place of work j. Using the work trip matrix calculated for each period in the transport submodel, net changes of employment of skill level q at place of residence i, ΔP^e_{qi}, can be inferred:

$$\Delta P^e_{qi}(t,t+1) = \sum_j \sum_m \frac{t_{q1ijm}(t)}{\sum\limits_{jm}\sum t_{q1ijm}(t)} \sum_s P^e_{sq}(t)[E^n_{sj}(t,t+1) - E^r_{sj}(t,t+1)] \quad (7)$$

where $P^e_{sq}(t)$ is the proportion of workers of skill level q in sector s at time t, and $t_{q1ijm}(t)$ are daily work trips (trip purpose g = 1) of workers of skill level q from i to j using mode m at the beginning of the period.

In the model, the four skill levels q correspond to the four income groups listed above, which means that workers of skill level q are assumed to belong to a household of type $h \in H_q$, i.e. to the set of household types belonging to income group q. With this assumption, for each residential zone i from $\Delta P^e_{qi}(t,t+1)$ a 4 x 4 matrix of transition rates between household income groups can be constructed and used for updating all household distributions of the zone including the occupancy matrix.

3.3 Market Clearing (Migration)

In this submodel, intraregional migration decisions of households are simulated as search processes in the regional housing market. Thus the migration submodel is at the same time the market clearing part of the housing market model.

Technically, the market clearing or migration submodel is a Monte Carlo micro simulation of a sample of representative housing market transactions The Monte Carlo technique, introduced into the social sciences by Orcutt et al. (1962), has attracted increasing interest recently as an analytical tool for studying spatial processes like transport choice, residential location, or the housing market characterized by high heterogeneity of demand and supply (see, for instance, Chapin and Weiss, 1968; Azcarate, 1970; Kain et al., 1976, Schacht, 1976; Kreibich, 1979; Oguri, 1980; Clarke, 1981). However, the use of the Monte Carlo technique in the Dortmung housing market model differs from other applications in a number of ways:

(1) In the Dortmund model, the Monte Carlo technique is used to model market transactions between households and landlords based on household and landlord preferences and supply characteristics. In this it differs from models like the NBER housing market model (Kain et al., 1976) or the GEWOS housing market model (Schacht, 1976), or the work done at Leeds (Clarke et al., 1979; 1980; Clarke, 1981), in which the Monte Carlo technique is used for simulating household and housing dynamics. That, however, is done in aggregate form in the Dortmund model, see Section 3.1. Conversely, the market clearing process of the NBER model is based on aggregate choice and optimization, while the GEWOS model applies heuristic priority rules for matching demand and supply. The Monte Carlo technique is used to stochastically model search processes in the classical work on residential location by Chapin and Weiss (1968) and the housing search models by Azcarate (1970) and Oguri (1980), however, only the first two use something like utility or attractiveness to guide the search, whereas Oguri constructs the search along predefined search sequences.

(2) In the application of the Monte Carlo technique in the Dortmund model the sampling of representative transactions is not exogenous, but is endogenized within the simulation. The reason for this uncommon practice lies in the assumption that the probability of a move is related to the difference between the utility of the old and the expected utility of the new dwelling and can thus not be determined before the actual market clearing. This means that the Dortmund model, unlike most other housing market models, does not use a "mover pool", into which prospective mover households are assembled prior to the market clearing. Instead, households willing to move are sampled during the market clearing simulation and are left in their old dwelling if the market transaction turns out to be unsuccessful. Of course, because the sampling is performed endogenously during the simulation, also the aggregation has to be performed endogenously.

(3) The Dortmund model does not use the list processing technique for the Monte Carlo simulation, as it is commonly done, but stores demand and supply information in matrix and hence necessarily aggregate form. The reason for this is that during the market clearing process a large number of searches have to be performed within the household and housing stock and that these searches use different search criteria at different times. Presently there are no efficient techniques available for searching in a random-order list. So the more structured matrix organization is preferred at the expense of

some loss of information.

The market clearing submodel starts from the following situation: All households and dwellings of all zones are aged by one simulation period, i.e. now have the time label of the end of the current simulation period. However, no household has yet moved to another dwelling. That is to say: All households have proceeded in their life cycle - they have become older, children may have been born, the family income may have increased -, but their dwellings are still the same or even have deteriorated. Moreover, the expectations of the households with respect to size, quality, and location of housing generally will have increased. It may be assumed that many households which were quite satisfied with their housing situation at the end of the last simulation period now are dissatisfied with it and are willing to improve it.

These households are the potential movers of the current market period. They are contained in the occupancy matrix $\underline{R}$. Besides, there are households without dwellings contained in the matrix $\underline{H}$ and vacant dwellings contained in the matrix $\underline{D}$. In addition, there are two H x 1 vectors of households specified by the Nordrhein-Westfalen model: the vector $\underline{H}^i$ containing households migrating into the region from elsewhere during the simulation period, and the vector $\underline{H}^o$ containing households migrating out of the region. Inmigrant households are treated just like households without dwelling, except that they do not come from a particular zone. Outmigrant households are of interest because they vacate a dwelling.

The matrices $\underline{R}$, $\underline{H}$, and $\underline{D}$, plus the vectors $\underline{H}^i$ and $\underline{H}^o$ are a complete representation of households and housing at the outset of the market clearing simulation. Of these, the matrix $\underline{H}$ and the vector $\underline{H}^i$ clearly represent the supply side. The matrix $\underline{R}$ represents some of both because of the linkage between housing supply and housing demand by vacant dwellings being put on the market with each move. But which ones of the households in $\underline{R}$ will actually move during the market period is not known at this time.

The decision behavior of the model actors in the housing market is controlled by utility indicators called <u>housing satisfaction</u>. The satisfaction of a household with its housing situation is represented by a utility function incorporating housing size and quality, neighborhood quality, location, and housing cost.

For use in the market clearing simulation, the four dimensions of housing satisfaction are stored in two matrices: For each combination of household type h, dwelling type k, and zone i, i.e. for each element of the three-dimensional occupancy matrix $\underline{R}$, an <u>index of housing satisfaction</u> u^h_{hki} is calculated as a weighted aggregate of three of the four dimensions:

$$u^h_{hki} = [u^h_{hk}]^{w^{hk}_h} \, [u^h_{hi}]^{w^{hi}_h} \, [u^{hp}_{hki}]^{w^{hp}_h} \tag{8}$$

where u^h_{hk} is the attractiveness of housing type k for household type h, u^h_{hi} is the attractiveness of zone i as a housing location for household type h, and u^{hp}_{hki} is the attractiveness of the rent or price of the dwelling in relation to the household's housing budget. The w^{hk}_h, w^{hi}_h, and w^{hp}_h are multiplicative weights adding up to unity. Both u^h_{hk} and u^h_{hi} are themselves multiattribute encompassing relevant attributes of the dwelling or the neighborhood:

$$u^h_{hk} = \sum_n w^{hk}_{hn} \, v^{hk}_{hn} [f^{hk}_{hn}(\underline{x}_k)] \tag{9}$$

$$u^h_{hi} = \sum_n w^{hi}_{hn} \, v^{hi}_{hn} [f^{hi}_{hn}(\underline{x}_i, \underline{u}^a_{qi})] \tag{10}$$

where n, n = 1, ..., N indicates attribute n and, as in equation (7), household type $h \in H_q$ belongs to income group q. The w^{hk}_{hn} and w^{hi}_{hn} are additive weights addin up to unity, the functions $v^{hk}_{hn}(.)$ and $v^{hi}_{hn}(.)$ are value functions mapping attributes to utility, and the functions $f^{hk}_{hn}(.)$ and $f^{hi}_{hn}(.)$ are generation functions specifying how to calculate attributes from one or more elements of vectors $\underline{x}_k$ or $\underline{x}_i$ or raw attributes of dwelling type k or zone i, and accessibility indices $\underline{u}^a_{qi}$ of zone i, see below. The housing price attractiveness u^{hp}_{hki} is calculated as

$$u^{hp}_{hki} = v^{hp}(p^h_{ki}/y^h_{hk}) \tag{11}$$

where p^h_{ki} is rent, or imputed rent in the case of owner-occupied dwellings, of dwelling type k in zone i, and y^h_{hk} is the monthly housing budget of household type h for this dwelling type. The housing budgets include housing allowances and other public subsidies and are therefore different for rented and owner-occupied dwellings.

The $\underline{u}^a_{qi}$ are household income-group specific vectors of accessibility indices describing the location of zone i in the region:

$$u^a_{qni} = \sum_j \sum_{m \in M_q} \frac{W_{nj} \exp(\beta^a_n \, u^t_{qijm})}{\sum_j \sum_{m \in M_q} W_{nj} \exp(\beta^a_n \, u^t_{qijm})} \, u^t_{qijm} \tag{12}$$

or

$$u^a_{qgi} = \sum_j \sum_{m \in M_q} \frac{t_{qgijm}}{\sum_j \sum_{m \in M_q} t_{qgijm}} \, u^t_{qijm} \tag{13}$$

Both accessibilities are expressed in terms of mean trip utility, i.e. as a weighted average of trip utilities u^t_{qijm} of trips from i to j using mode m for household income group q, where the term trip utility denotes a measure (calculated in the transport submodel) which aggregates monetary and non-monetary costs and benefits of trips in terms of utility. In the first form of accessibility, the weights are potential trips to activities of facilities W_{nj} of type n in j, the second accessibility uses daily trips of purpose g calculated in the transport submodel, t_{qgijm}, as weights. The set M_q includes all transport modes accessible to household income group q, depending on its car ownership level. The β^a_n, like all lower case Greek letters in the following equations, are parameters to be estimated.

Obviously, in the index of housing satisfaction, only average zonal accessibilities like the ones in (12) and (13) can be included. Therefore an additional relational location measure is calculated for each pair of zones. The accessibility measure $u^m_{qii'}$ describes the locational attractiveness of a zone i' as a new housing location for a household of type income group q now living in zone i and working in any of the zones near i, hence it is called "migration utility":

$$u^m_{qii'} = \left[\sum_j \sum_{m \in M_q} \frac{t_{q1ijm}}{\sum_j \sum_{m \in M_q} t_{q1ijm}} \, u^t_{qi'jm}\right]^{w^c_q} \left[\sum_{m \in M_q} \frac{t_{q3ii'm}}{\sum_{m \in M_q} t_{q3ii'm}} \, u^t_{qii'm}\right]^{w^s_q} \tag{14}$$

where t_{qgijm} and u^t_{qijm} are again daily trips of purpose g and the corresponding trip utilities. The first part of the expression is the expected utility of a work trip (g = 1) from the new housing zone i' to all possible old work zones j after the move, the second part evaluates the utility of a social or service trip ($g = 3$) between the old and the new housing zone. The w_h^c and w_h^s are multiplicative weights adding up to unity. For a more extended discussion of migration utilities see Wegener (1983).

With the matrices $\underline{R}$, $\underline{H}$, $\underline{D}$, $\underline{u}^h$, and $\underline{u}^m$, and the migrant household vectors $\underline{H}^i$ and $\underline{H}^o$ all necessary information is available to enter the simulation of the market clearing process.

The basic unit of the simulation in this submodel is the market transaction. A market transaction is any operation by which a migration occurs, i.e. a household moves into or out of a dwelling or both. There are two transaction types or ways to start a market transaction: A household decides to look for a dwelling ("dwelling wanted"), or a landlord decides to offer a dwelling ("dwelling for rent or sale"). In either case the transaction may result in different kinds of migration: The household may leave the region ("outmigration") or enter it ("inmigration"), or currently be without a dwelling ("new household or forced move"), or occupying one ("move"). For the landlord offering a dwelling only the last three migration types are of interest.

The model starts by selecting a transaction type and a migration type. For the first transaction to be simulated it is assumed the "dwelling wanted" and "dwelling for rent or sale" are equally likely to occur. The migration type is selected in proportion to the number of migrations to be completed of the first three migration types, i.e. the totals of $\underline{H}^o$, $\underline{H}^i$, and $\underline{H}$. For the fourth or "move" migration type a tentative estimate of the number of moves as a portion of the total of matrix $\underline{R}$ must be provided.

Once the transaction type and the migration type have been determined, the remaining parameters of the transaction are selected. A transaction has been completely defined if the following five parameters are known:

- h household type
- k old housing type
- i old zone
- k' new housing type
- i' new zone

A move, for instance, is a migration of a household of type h which occupies a dwelling of type k in zone i into a dwelling of type k' in zone i'. Not all five parameters are required for all migration types: Obviously, no k can be specified for households without a dwelling, nor can k and i for inmigrant households. Of outmigrant households only h, k, and i are of interest.

The Monte Carlo simulation of a market transaction has four phases: a sampling phase, a search phase, a choice phase, and an aggregation phase:

- In the sampling phase, a household looking for a dwelling or a landlord looking for a tenant is sampled for being simulated.
- In the search phase, the household looks for a suitable dwelling, or the landlord looks for a suitable tenant.
- In the choice phase, the household decides whether to accept the negotiated dwelling.
- In the aggregation phase, all necessary changes of households and dwellings resulting from the transaction, multiplied by the sampling factor, are performed.

In the sampling phase, sampling is done either pro rata or controlled through multinomial logit functions. For instance, in the case of a household looking for a dwelling ("dwelling wanted"/"move"), a household type is selected pro rata from all household types. However, it is assumed that households which are less satisfied with their present dwelling are more likely to start looking for a new one. This is expressed by a logit function in which the index of housing satisfaction figures negatively. To give an example,

$$p_{k|hi}(t') = \frac{R_{hki}(t')\exp[-\alpha_h^h\, u_{hki}^h(t)]}{\sum\limits_k R_{hki}(t')\exp[-\alpha_h^h\, u_{hki}^h(t)]} \tag{15}$$

is the probability that, at time t' between t and t+1, of all households living in zone i, one occupying a dwelling of type k will be sampled for simulation. In this equation, $R_{hki}(t')$ is the number of such households at time t', and $u_{hki}^h(t)$ is their housing satisfaction as defined in (8). Because of its negative coefficient, this function will result in a higher proportion of dissatisfied households entering the housing market than would result from pro rata sampling. Note that the index of housing satisfaction, like all utilities used in the model, carries the time label t of the beginning of the simulation period, i.e. is lagged, whereas all variables with the time label t' are continuously updated during the simulation.

In the search phase, search decisions of households and landlords are modeled. It is therefore entirely controlled by multinomial logit choice functions. To continue the example of the moving household,

$$p_{i'|hki}(t') = \frac{\sum_{k'} D_{k'i'}(t')\exp[\beta_h^m u_{qii'}^m(t)]}{\sum_{i'}\sum_{k'} D_{k'i'}(t')\exp[\beta_h^m u_{qii'}^m(t)]} \qquad (16)$$

is the probability that the household searches in zone i' for a new dwelling, and

$$p_{k'|hkii'}(t') = \frac{D_{k'i'}(t')\exp[\gamma_h^h u_{hk'i'}^h(t)]}{\sum_{k'} D_{k'i'}(t')\exp[\gamma_h^h u_{hk'i'}^h(t)]} \qquad (17)$$

is the probability that it inspects a dwelling of type k' there before making a choice. In these two equations, $R_{hki}(t')$ again is the number of households of type h living in a dwelling of type k in zone i at time t', and $D_{k'i'}(t')$ is the number of vacant dwellings of type k' in zone i' at that point in time. The $u_{hki}^h(t)$ and $u_{qii'}^m(t)$ are the two utilities defined in (8) and (14) expressing the attractiveness of a dwelling or a zone for a household considering a move. The first of the two choice functions indicates that the household is likely to start its search in a zone neither too distant from its present residence zone nor from its place of work. The second choice function indicates how the household searches among the vacant dwellings of that zone and selects one for inspection based on the expected housing satisfaction it associates with each of them. The effect of this function is that in general only dwellings having a good chance of being acceptable for the household are selected.

In the choice phase, the actual migration decision is made. This is no question for outmigrant households, they do migrate. All other households compare their present housing situation with the situation they would gain if they accepted the transaction. It is assumed that they behave as satisficers, i.e. that they accept if they can improve their housing situation by a significant margin:

$$p_{a|hkik'i'}(t') = \begin{cases} 1 & \text{if } u^h_{hk'i'}(t) - u^h_{hki}(t) \geq a_h(t') \\ \\ 0 & \text{if } u^h_{hk'i'}(t) - u^h_{hki}(t) < a_h(t') \end{cases} \tag{18}$$

where $p_{a|hkik'i'}(t')$ is the probability that a household of type h in this situation accepts, and $a_h(t')$ is its current aspiration level, i.e. the amount of improvement necessary to make it move.

The value of $a_h(t')$ is assumed to depend on the search experience of all households of type h, i.e. go up with each successful and down with each unsuccessful search of a household of type h. In other words, households are assumed to collectively adapt their aspiration levels to supply conditions on the market:

$$a_h(t'') = \begin{cases} a_h(t') + \Delta a & \text{if } p_{a|hkik'i'}(t') = 1 \\ \\ a_h(t') - \Delta a & \text{if } p_{a|hkik'i'}(t') = 0 \end{cases} \tag{19}$$

where $a_h(t'')$ is the new value of $a_h(t')$ updated after each market transaction, and Δa is an increment/decrement much smaller than the initial value of $a_h(t)$ to prevent excessive oscillations of aspiration levels.

The <u>aggregation</u> phase concludes a successful transaction. If the household accepts the dwelling, all necessary changes in $\underline{R}$, $\underline{H}$, $\underline{D}$, $\underline{H}^i$, and $\underline{H}^o$ are immediately performed. Dwellings vacated with a move or an outmigration reappear in the matrix $\underline{D}$ and are thus again released to the market. All changes caused by a transaction are multiplied by the sampling factor before execution, i.e. sampling and re-aggregation are linked to maintain internal consistency of all stock variables.

If the household does not accept the dwelling, it enters another search phase, but after a number of unsuccessful attempts it abandons the idea of a move. If the household declines, and the transaction type was "dwelling for rent or sale", the landlord tries to find another household, but it is not assumed that he reduces the rent during the market period.

The sequence of selection steps used with each combination of transaction type and migration type is shown in Tables 2a and 2b. For each step, the choice set is shown together with the utility of each choice alternative available for selection. To derive the conditional choice probabilities, multinomial choice functions similar to the ones in (15) - (17) can be constructed from Tables 2a and 2b. Where the utility is one, selection from the choice set occurs <u>pro rata</u>.

Migration type	Choice parameter	Choice set	Utility
outmigration	h	H_h^o	1
	i	$\sum_k R_{hki}$	$\sum_k -u_{hki}^h$
	k	R_{hki}	$-u_{hki}^h$
inmigration	h	H_h^i	1
	i'	$\sum_{k'} D_{k'i'}$	$u_{hii'}^m$
	k'	$D_{k'i'}$	$u_{hk'i'}^h$
new household/ forced move	h	$\sum_i H_{hi}$	1
	i	H_{hi}	1
	i'	$\sum_{k'} D_{k'i'}$	$u_{hii'}^m$
	k'	$D_{k'i'}$	$u_{hk'i'}^h$
move	h	$\sum_i \sum_k R_{hki}$	1
	i	$\sum_k R_{hki}$	$\sum_k -u_{hki}^h$
	k	R_{hki}	$-u_{hki}^h$
	i'	$\sum_{k'} D_{k'i'}$	$u_{hii'}^m$
	k'	$D_{k'i'}$	$u_{hk'i'}^h$

Table 2a. Choice sequences: Dwelling wanted.

Migration type	Choice parameter	Choice set	Utility
outmigration	-		
	-		
	-		
inmigration	k'	$\sum_{i'} D_{k'i'}$	1
	i'	$D_{k'i'}$	1
	h	H_h^i	$u_{hii'}^m$
new household/ forced move	k'	$\sum_{i'} D_{k'i'}$	1
	i'	$D_{k'i'}$	1
	h	$\sum_i H_{hi}$	$u_{hk'i'}^h$
	i	H_{hi}	$u_{hii'}^m$
move	k'	$\sum_{i'} D_{k'i'}$	1
	i'	$D_{k'i'}$	1
	h	$\sum_i \sum_k R_{hki}$	$u_{hk'i'}^h$
	i	$\sum_k R_{hki}$	$u_{hii'}^m$
	k	R_{hki}	$-u_{hki}^h$

Table 2b. Choice sequences: Dwelling for rent or sale.

After the successful completion of a transaction, the next transaction of the same transaction type is initiated. After each unsuccessful transaction, the model switches the transaction type, i.e. from "dwelling wanted" to "dwelling for rent or sale" or vice versa. By this mechanism, always the more efficient search mode is favored by the model: If supply is large, household have more choice options and start to search more frequently; if however supply is small, landlords have no problems to find a household for their dwelling, so more market transactions are started from the landlord side.

The market process comes to an end when there are no more households considering a move. It is assumed that this is the case when a certain number of transactions has been rejected. This number has to be determined by calibration to match the number of migrations produced by the model with the number of migrations observed in the region.

During the market clearing simulation, a transaction protocol keeping track of all successful transactions is written. After the simulation, the protocol serves to calculate migration flows by household type between dwelling type and zones, and to perform all migration-induced changes of the age, household, and labor force distributions of the zones.

3.4 Public Housing Construction

The model user may specify major changes of the housing stock in particular zones and years exogenously. This device is useful for entering large public subsidized housing construction and rehabilitation projects.

3.5 Private Housing Construction

The submarkets of the housing construction submodel are the housing types of the aggregate (30-type) housing classification in the zones, i.e. each combination of housing type and zone represents a separate submarket. Private housing investments can be either maintenance/upgrading or new construction investments.

a) Maintenance/Upgrading

Landlords are assumed to invest into their existing housing stock if they can expect to raise their profit by doing so. The proportion of dwellings upgraded in each period is calculated for each dwelling type in each zone as a function of the expected rent increase in that submarket after improvement. As the eventual rent increase is not known at this time, the landlords employ a simple rent expectation model based on vacancy rates in the submarket at the beginning of the simulation period. The elasticity function controlling landlord investment behavior is exogenous.

Maintenance/upgrading and filtering (see Section 3.1) work in the opposite direction. Filtering means that a proportion of dwellings drops down the quality scale by one level, maintenance/upgrading means that this is offset (maintenance) or more than offset (upgrading) by repair or improvement. The net effect of maintenance/upgrading and filtering may result in an overall deterioration or improvement of the housing quality in a zone.

b) New Housing Construction

Housing investors are assumed to invest into new housing construction if they can expect to have higher profits by doing so than by investing into something else. The supply of new dwellings thus provided in each submarket is calculated for each period as a function of the expected rent increase in the submarket using a similar rent expectation model as in the maintenance/upgrading submodel. However, the new supply provided in a submarket is not necessarily built in that submarket zone, as there may be no suitable vacant land available, but is aggregated to dwelling types and allocated to vacant residential land in the whole region by the following multinomial logit model:

$$D^n_{k\ell i}(t,t+1) = \frac{C^d_{k\ell i}(t')\exp[\gamma^d_k\, u^d_{k\ell i}(t)]}{\sum\limits_i \sum\limits_\ell C^d_{k\ell i}(t')\exp[\gamma^d_k\, u^d_{k\ell i}(t)]}\; D^n_k(t,t+1) \tag{20}$$

where $D^n_k(t,t+1)$ is the expected supply of new dwellings of type k provided in the period, and $D^n_{k\ell i}(t,t+1)$ is the number of dwellings of type k built on land use category ℓ in zone i. $C^d_{k\ell i}(t')$ is the capacity of that vacant land for dwellings of type k at time t'. Note that $C^d_{k\ell i}(t')$ is successively reduced during the simulation period by land uses with similar land requirements. The utility $u^d_{k\ell i}$ expresses the attractiveness of land use category ℓ in zone i for dwellings of type k as seen by the housing investors:

$$u^d_{k\ell i} = [u^d_{k\ell}]^{w^{d\ell}_k}\; [u^d_{ki}]^{w^{di}_k}\; [u^{dp}_{k\ell i}]^{w^{dp}_k} \tag{21}$$

where $u^d_{k\ell}$ is the attractiveness of land use category ℓ for housing type k, u^d_{ki} is the attractiveness of zone i as a location for housing type k, and $u^{dp}_{k\ell i}$ is the attractiveness of the land price of land use category ℓ in zone i in relation to the expected rent or price of the dwelling. The $w^{d\ell}_k$, w^{di}_k, and w^{dp}_k are multiplicative weights adding up to unity. The component utilities are similarly constructed as the components of the housing utility u^h_{hki} in (8). Like all utilities in the model, $u^d_{k\ell i}$ is used in lagged form as calculated at time t.

Dwellings built during a simulation period utilize land immediately, but become available to the housing market only in the subsequent period.

3.6 Price Adjustment

Price adjustment in the model is delayed, i.e. is performed at the end of each simulation period using information generated during the period, but becomes effective only in the subsequent period to reflect the inertia inherent in regulated markets.

a) Housing Prices/Rents

The price of housing is represented in the model in the form of monthly rent per dwelling unit by type, in the case of owner-occupied houses or flats in the form of imputed rents.

The model contains three different mechanisms to update housing prices/ rents from period to period:

- The first rent submodel inflates all rents according to a regionwide, exogenously specified rent inflation rate.
- The second rent submodel adjusts rents in particular submarkets whenever new or modernized dwellings are released to the housing market. Upgraded dwellings are more expensive than before, and new dwellings are larger and more expensive. The resulting submarket rent is an average of old and new rents.
- The third rent submodel modifies the results of the first two in response to demand observed on the housing market. For each submarket, i.e. each combination of dwelling type and zone, it increases or decreases the inflated rent as a function of the vacancy rate in the submarket after the housing market simulation following an exogenously specified elasticity function.

No attempt is made to establish equilibrium rents on the housing market within a simulation period. Rents remain fixed during the market clearing process and are adjusted only for the following period.

b) Land Prices

The model contains two mechanisms to update land prices from period to period:

- The first land price submodel inflates all land prices according to a regionwide, exogenously specified land price inflation rate.
- The second land price submodel modifies the results of the first one in response to demand observed on the land and construction market. For each land use category in each zone, it increases or decreases the inflated land price as a function of the percentage of the total supply of land (cleared or vacant) that was actually developed and utilized during the simulation period following an exogenously specified elasticity function.

No attempt is made to establish equilibrium land prices within a simulation period. Land prices remain fixed during the whole land use allocation process and are adjusted only for the subsequent period.

4. Model Data and Calibration

The main data sources for the housing market model were tapes of the 1968 housing census and the 1970 population and employment census specially prepared for this project by the City of Dortmund. They were the basis for establishing the disaggregate (120-type) distribution of households and housing and of the occupancy matrix of each zone.

Constructing the 120-type household distribution for the base year 1970 presented no particular problems as far as they could be aggregated from the census tapes containing individual data. However, for the communities outside of Dortmund, for which such tapes were not available, biproportional estimates based on onedimensional distributions taken from published statistical tables had to be made. A special estimation technique had to be developed to substitute the income information not contained in the census data. By this technique, each household was associated with one of the four income groups depending on the employment status and completed education of its head, both of which informations were available on the tapes (see Gnad and Vannahme, 1981).

Base year data of the housing stock were taken from the 1968 housing census. As with the household data, tapes containing information on a dwelling-by-dwelling basis were available for Dortmund, while biproportional estimates from marginal distributions had to be made for the communities outside of Dortmund. All information needed to establish the 120-type distribution of each zone was contained on the tapes. However, the fourth dimension constituting a dwelling type, housing quality, had to be aggregated from a number of dwelling attributes (see Gnad and Vannahme, 1981).

A special problem was encountered in the construction of the base year occupancy matrix of each zone. The 1968 housing census contained detailed housing information, but only very limited information on households. The 1970 census, however, contained detailed household, but no housing information. The problem was to match both kinds of census, although they were 18 months apart in time. The problem was solved by first generating for each zone a household-housing matrix from the 1968 data, and then "blowing it up" to match the 1970 household distribution (see Gnad and Vannahme, 1981).

Calibration of the model involves the choice of the most suitable values (and their trajectories over time) of the following groups of model parameters:

- demographic parameters,
- household parameters,
- housing parameters,
- land use parameters,
- monetary parameters,
- preference parameters.

The first five of these are expressed in terms of physical-technical or monetary quantities or rates, are therefore always dimensioned (e.g., deaths per year, DM per square meter, etc.) and, at least in principle, measurable during an observation period. The last group of parameters, however, express human preferences, i.e. serve to translate attributes of the physical world into utilities; they are altogether undimensioned and for practical and theoretical reasons extremely difficult to measure.

Below, the methods applied to determine the values of the above parameters are summarized. Parameters needed for submodels only referenced in this chapter, such as the economic or transport submodel, are not included in the discussion.

a) Demographic Parameters

The demographic parameters consist essentially of age-group specific birth rates and age-group and sex specific survival rates needed for the household projection submodel (see Section 3.1). Birth rates are disaggregated by nationality (native, foreign) and type of zone (urban, rural) to account for national and regional differences in reproductive behavior. Both kinds of rates are well documented for the past and can be retrieved from published statistical tables. However, there is always some speculation connected with the extrapolation of birth rates into the future. Quite recently, the falling tendency of birth rates in the Federal Republic has been reversed. Will this upswing persist or will it remain a minor perturbation? Assumptions about future birth rates are crucial because minor differences in birth rates will have relatively large impacts on the size distribution and thus housing demand of households already in the near future.

b) Household Parameters

The household parameters include the rates necessary for estimating the probability of events such as "marriage", "divorce", "new household of child", or "relative joins household" for each household type. Such data are nonexistent or only available by age group, but not by household type, as marriage and divorce rates. So the parameter values used had to be synthesized from fragmentary qualitative information. However, the only alternative to this procedure would have been to ignore these kinds of household transformations altogether, which is no real alternative in a model based on household decisions. Even more uncertain is the projection of these parameters into the future: Will the tendency of falling marriage rates and rising divorce (and remarriage) rates continue? Will young persons continue to separate from their parents' household as early as possible in the face of growing youth unemployment? Will the three-generation family continue to gradually disappear? The answers to these questions are critical for the projection of the future household composition of households and thus for forecasting housing demand.

c) Housing Parameters

The housing parameters include the rates needed for driving the housing filtering submodel (see Section 3.1). Rather broad assumptions about the rate of deterioration and demolition of buildings had to be made, because these occur largely unrecorded by any statistics. In addition, the housing parameters include information on average size (in square meters) of newly constructed dwellings by type, which have considerably increased in the past and are likely to continue to do so for quite some time.

d) Land Use Parameters

The land use parameters include the technical coefficients necessary to calculate the land requirements of residential buildings at a given density. They are used to determine the number of dwellings by type that can be erected on vacant land of a certain land use category and density in the zoning plan, see Section 3.5. These parameters are well defined in engineering terms and can be expected to remain stable over time.

e) Monetary Parameters

The monetary parameters contain all information necessary for periodically updating household incomes (see Section 3.2) and housing and land prices (see Section 3.6).

On the household side, this includes household budgets by household type for housing, transport, and other living expenditures, as well as all sorts of public subsidies relevant for housing choice decisions such as children and housing allowances for all households and, in addition, various kinds of tax benefits and special loans and subsidies for owner-occupier households in accordance with the housing legislation of the Federal Republic.

The household income parameters also include maximum income levels for eligibility of households of different sizes for public housing units. Specifying these parameters, in general, presented no problems, except that the immense number and variety of relevant legal regulations required some simplification and hence loss of detail.

On the housing side, the monetary parameters include the various cost components that make up the cost of a new or improved dwelling such as construction or improvement costs by dwelling type and a discounting factor used to convert them into monthly rents. This kind of cost information could be extracted from annually published statistics. However, the elasticity function presently used to model the price adjustment behavior of landlords between market periods was based entirely on plausibility assumptions, as no information on such behavior exists for the region. This may seem more critical than it really is, as the rent adjustment submodel employs an intendedly "primitive" rent expectation model of the landlord and is not designed to produce equilibrium rents, but asking rents for the next market period that may be corrected by the results of that period.

The same applies to the elasticity function used in the land price adjustment submodel. Land prices produced in the land price adjustment submodel are not meant to be equilibrium land prices, but asking prices for the next simulation period and subject to correction after that period. However, the long-term reasonableness of land prices is essential for the model because land prices not only influence the housing construction submodel, but also represent a sizable component of the price of new housing.

All monetary parameters on the demand and on the supply side of the housing market are inflated by appropriate inflation rates at the beginning of each period. Much of the dynamics of the model is due to the differences between those inflation rates. For instance, if housing costs rise faster than transport costs, households will tend to choose more peripheral housing locations with lower rents, and vice versa. Therefore, the careful projection of inflation rates for incomes, housing and construction costs, land prices, and transportation was an important part of the calibration.

f) Preference Parameters

The preference parameters are the parameters of the preference or choice functions of the model. On the demand side, these are choice functions of households looking for a dwelling such as equations (15) - (17) and all equations implied by Tables 2a and 2b. On the supply side, these are choice functions of housing investors looking for vacant residential land such as equation (20).

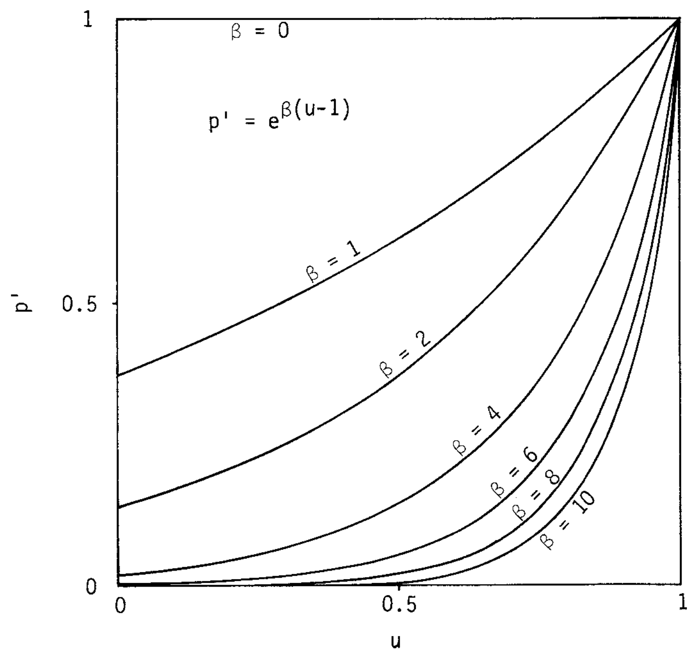
By nature, the choice functions are different for different decision maker groups or submarkets, i.e. household types on the demand side and dwelling types on the supply side. They differ in the number, selection, and specification of attributes and in the way the attributes are aggregated. The number of attributes contained in the choice functions varies between 2 and 27, and the total number of choice functions is 232 or 482, depending on the way of counting.

Statistical estimation of such a vast number of multiattribute choice functions requires a large sample of data on observed housing market transactions, either disaggregate or classified in the way household types, dwelling types, and zones are defined in the model. Such data do not exist for the study region. In fact the model was not designed to match any existing set of data, but was constructed only following considerations of conceptual consistency and substantive integrity. If this precluded statistical estimation, model structure was always considered to be more important than estimatibility.

To make such a non-survey modeling strategy feasible, the parameters of the choice functions were standardized in the following way in order to make them comparable over applications and amenable to discussion and judgement.

- Raw attributes of the choice alternatives are converted into utility-relevant attributes by generation functions f(.), see equations (9) and (10).
- Utility-relevant attributes are mapped to a standardized utility scale, say between zero and one, by value functions v(.), see equations (9) - (11).
- The resulting component utilities are aggregated to higher level utilities using weights w. Aggregation is normally additive, as in equations (9) and (10).
- On the highest aggregation level, multiplicative aggregation is used to exclude alternatives from the choice set that violate a restriction, i.e. are unacceptable with respect to a very important attribute, see equations (8), (14), and (21).
- In each aggregation function, weights w add up to unity. This implies that also all aggregate utilities are standardized, irrespective of whether the aggregation was additive or multiplicative.
- For each alternative, only the most aggregate standardized utility is entered into the actual choice function, see equations (15) - (17) and (20). This implies that the exponent parameters of the choice functions, α, β, γ, etc. are also standardized.

The last standardization effect is of interest because it illuminates some of the behavioral interpretations of the logit model. The exponent parameter, say β, can be interpreted as a measure of homogeneity of the preferences hidden behind the mean utility (see Anas, 1983) or as a kind of elasticity indicating how much the choice probability of an alternative is affected by a change in its utility. Both interpretations lead to the same consequences. Small values of β indicate diversity in preferences and/or alternatives, accordingly the choice pattern will be dispersed; large values of β indicate homogeneity of preferences and/or alternatives, accordingly the choice pattern will tend to maximize utility (Brotchie et al., 1980). If β is standardized in the above manner, each value of β implies a certain slope of the exponential function and thus a particular point between indifference and optimization. Figure 2 illustrates this by showing standardized exponential functions for various levels of β mapping utility into relative choice probabilities (odds) and the corresponding market shares in the case of binomial choice.



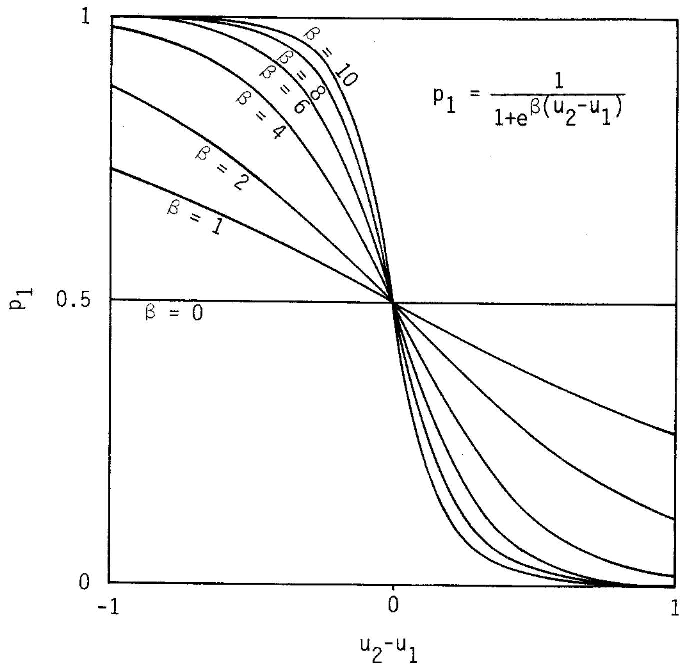


Figure 2. Standardized exponential functions (top) and corresponding market shares in the binomial case (bottom).

The standardization scheme described above should be seen in contrast to the usual way of applying the multinomial logit model. Normally, the α, β, γ, etc. and the w, v(.), and f(.) are lumped together into one vector of additive coefficients that can be statistically estimated by maximizing its likelihood function (see, for instance, Anas, 1982). While this is dictated by the estimation technique, it remains unfortunate as it obscures the sequence of transformations the data have to undergo rendering the estimated coefficients virtually uninterpretable. This, however, makes the model totally dependent on survey data and seriously restricts its transferability from one application to the other.

Moreover, the additive form of utility in the exponent can lead to serious misspecification where clearly interactions between attributes exist, and this is, by definition, the case where very important attributes act as restrictions, i.e. exclude an alternative from the choice set if not satisfied.[1]

Using the above standardizations, for each household type a preference structure consisting of generation functions, value functions, and several hierarchical levels of weights was specified. Figure 3 shows a typical preference hierarchy of a household as it is used to calculate the index of housing satisfaction, see equations (8) - (13). Similar preference hierarchies were constructed for housing investors looking for vacant residential land.

Statistical estimation techniques were not used to select the attributes and estimate the parameters of the value and aggregation functions. All value functions and weights were determined entirely by judgment, inferences, analogies, and careful checking of plausibility taking into account the changing housing and locational needs of households of different age, number of children, and nationality. The empirical foundations of this informal way of model calibration included published reports on the numerous surveys of regional and urban housing markets conducted in West Germany in recent years, some of them covering parts of the study region (see, among others, Infas, 1978; Landwehrmann and Kleibrink, 1978; Landwehrmann and Körbel, 1980; Hosemann et al., 1980). Particular care was taken to as realistically as possible model the economic situation of different household types with respect to their housing expenditures, housing allowances, and other housing budget components, especially for owner-occupier households.

In addition, it was attempted to assess which elements of the preference functions were most likely to change in the future. For two attributes, housing floorspace and number of rooms, time-dependent value functions were specified to account for growing housing floorspace and dwelling unit size standards.

The result of the exercise were the u^h_{hki} and $u^d_{k\ell i}$ of equations (8) and (21), respectively. In a final step, the exponent parameters of the

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[1] As an example consider the case of a hypothetical household whose utility function contains only the attributes "housing quality" and "accessibility", both with equal weight. For this household, in the additive model an excellent dwelling at an unacceptable location and an unacceptable dwelling at an excellent location both have a utility of 0.5, while in fact both should have one of zero. This case is handled correctly by the multiplicative model, which behaves much like the additive model in less extreme cases.

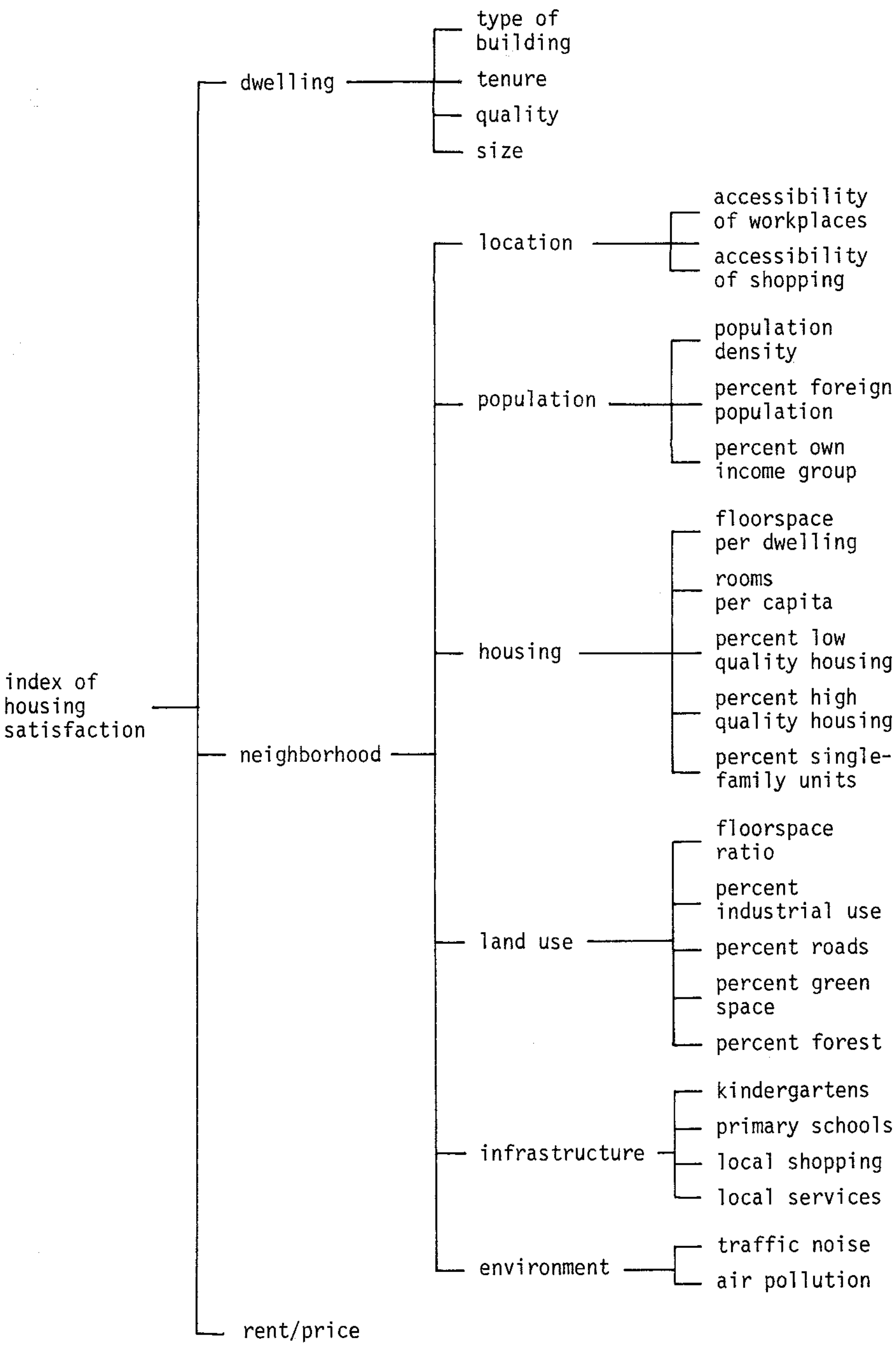

Figure 3. The index of housing satisfaction.

choice functions, i.e. the α, β, γ, etc. were determined. They are in fact the only parameters of the housing market model estimated statistically. One illustrative example is given below.

Equation (17) models the choice of a dwelling of type k' in zone i' by a household of type h. In order to estimate the parameter γ_h^h in (17), the cross-sectional information about past housing choice decisions contained in the base year occupancy matrix was used. The following assumptions were made:

- All households are looking for a dwelling.
- All dwellings, occupied or vacant, are available supply.

Now the question was asked: Which values of γ_h^h will best reproduce the base-year occupancy pattern? That is at the same time a test if the index of housing satisfaction is capable of explaining the observed choice pattern. Note that rent/price utility is included in the index of housing satisfaction.

To limit the number of parameters to be estimated, the estimation was performed for household income groups q rather than for household types h. To find the best-fit value of γ_q^h, the likelihood function of equation (17), or rather its logarithm, is maximized (see, for instance, Wegener and Graef, 1982):

$$\max_{\gamma_q^h} L^*(\gamma_q^h) = \sum_{h \in H_q} \sum_{k'} R^o_{hk'i'} \ln \left\{ \frac{\bar{D}_{qk'i'} \exp[\gamma_q^h u^h_{hk'i'}(t)]}{\sum_{k'} \bar{D}_{qk'i'} \exp[\gamma_q^h u^h_{hk'i'}(t)]} \right\} \tag{22}$$

where $\underline{R}^o$ is the original occupancy matrix and H_q is the set of household types h belonging to household income group q. $\bar{D}_{qk'i'}$ is the assumed housing supply for household income group q defined as

$$\bar{D}_{qk'i'} = \sum_{h \in H_q} R^o_{hk'i'} + D_{k'i'} \tag{23}$$

i.e. as the total of dwellings now occupied by households of income group q and vacant dwellings.

The maximum of the loglikelihood function (22) is where its gradient is zero:

$$\frac{dL^*(\gamma_q^h)}{d\gamma_q^h} = \sum_{h\in H_q} \sum_{k'} R^o_{hk'i'} \, u^h_{hk'i'}$$

$$- \sum_{h\in H_q} [\sum_{k'} R^o_{hk'i'}] \sum_{k'} p_{k'|hi'} \, u^h_{hk'i'} = 0 \qquad (24)$$

where $p_{k'|hi'}$ is the conditional probability that a dwelling of type k' is selected as defined by equation (17).

The estimation results for household income group 2 by zone are presented in Table 3. The second column in Table 3 shows the estimated parameter γ_2^h. It can be seen that it is relatively stable over zones, which suggests that in the model the same parameter value may be used for all zones. As the utilities $u^h_{hk'i'}$ used for the estimation were standardized as described above, also the estimated parameter values are standardized and can be compared with the parameter levels of Figure 2. It can be seen that they are located in the lower middle range of the sample exponential functions of Figure 2. This is quite plausible and well reflects the lack of information and the large number of choice restrictions existing on the housing market.[2]

The third and fourth columns of Table 3 show two goodness-of-fit measures for the 270 possible combinations of household type and dwelling type for household income group 2: the r-square statistic and the mean absolute percentage error MAPE defined as

$$\mathrm{MAPE} = \frac{\sum_{h\in H_q} \sum_{k'} |R^o_{hk'i'} - R_{hk'i'}|}{\sum_{h\in H_q} \sum_{k'} R^o_{hk'i'}} \, 100 \qquad (25)$$

where $\underline{R}^o$ is the observed and $\underline{R}$ the predicted occupancy matrix. The last two columns contain the same two measures for aggregate demand, i.e. for choices aggregated to the 30 dwelling types.

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[2] By comparison, the parameters of the choice functions in the transport submodel range between about 6 and about 15 depending on the purpose of travel.

Zone	γ_2^h estimated	Choices[a]		Demand[b]	
		r^2	MAPE[c]	r^2	MAPE[c]
1	3.4326	0.9055	29.10	0.9895	8.92
2	3.3995	0.8822	31.49	0.9898	8.07
3	3.3248	0.9295	25.95	0.9894	10.34
4	3.6790	0.8210	37.90	0.9891	8.88
5	3.8388	0.8170	45.50	0.9908	9.51
6	3.9621	0.8276	40.12	0.9959	9.15
7	3.8896	0.7989	39.10	0.9832	10.31
8	4.0454	0.8361	34.10	0.9856	10.84
9	3.6706	0.8156	34.86	0.9823	10.15
10	3.6805	0.8156	36.24	0.9834	9.03
11	3.1170	0.8232	39.80	0.9917	7.48
12	3.2260	0.8283	36.34	0.9857	8.52
13	3.3882	0.8014	36.72	0.9908	8.39
14	3.3131	0.7732	48.20	0.9913	8.58
15	3.4609	0.8161	36.67	0.9892	8.97
16	2.7943	0.7812	45.71	0.9897	6.63
17	2.7330	0.7761	42.41	0.9902	6.60
18	3.1226	0.7954	49.65	0.9956	7.68
19	3.1006	0.7784	41.54	0.9874	8.48
20	3.3150	0.8252	33.40	0.9824	8.68
21	2.8576	0.8035	36.60	0.9818	7.69
22	3.3903	0.8158	34.53	0.9852	8.22
23	3.5826	0.8377	33.71	0.9894	8.64
24	3.5098	0.8221	34.97	0.9822	8.22
25	3.6614	0.8181	37.42	0.9882	7.74
26	3.8904	0.8290	36.03	0.9810	9.59
27	4.0900	0.8462	31.89	0.9821	10.26
28	3.9180	0.8399	32.24	0.9835	9.41
29	3.6553	0.8281	32.66	0.9840	9.51
30	3.3028	0.8095	35.98	0.9860	8.50

[a] n = 270 [b] n = 30 [c] mean absolute percentage error

Table 3. Calibration results: Choice of dwelling, household income group 2, all zones.

The goodness-of-fit measures indicate that the correspondence between the observed and the predicted occupancy matrix is high in aggregate terms and quite good at the disaggregate level. The prediction errors indicated by the MAPE statistic at the level of choices are not surprising given the frictions and mobility costs on the housing market, which make the allocation of dwellings to households far from user-optimal. In this respect the results obtained do not differ from those of other empirical studies of cross-sectional housing market data (see, for instance, Gabriel et al., 1980). However, if the choices are aggregated to dwelling types, even the MAPE statistic indicates only minor prediction errors, which are certainly not larger than those of many empirical studies based on much better data. It can be concluded that the index of housing satisfaction, despite its being determined by intuitive reasoning only, seems to capture a considerable part of the variation in the housing choice of households.

Still it has to be admitted that the above calibration exercise is completely static and does not really reflect the actual choice situations of households on the housing market as they are modeled in the Monte Carlo simulation: as searches in a sequence of searchers. However, it is doubtful if search behavior can be reconstructed from survey data at all. Even the most elaborate survey on actual choices, if it were available, would only report the <u>result</u> of the search, but not the search <u>process</u> itself with its succession of aspirations and frustrations the Monte Carlo simulation tries to replicate. So in a practical sense the model is unestimable.

The way out of this situation is to <u>validate</u> the model where it cannot be <u>calibrated</u>. Validation in place of calibration has two great advantages. The most important one is that it saves the data normally used for calibration for the validation, and that may be crucial where few data exist. The second advantage is that it allows to test the model holistically, i.e. in all its dimensions, including <u>time</u>, whereas calibration usually proceeds one equation at a time and thus ignores feedbacks that may be essential. On the other hand, validation without calibration has the great disadvantage that it does not <u>prove</u> anything about a model, but at best increases its credibility by a certain margin.

During the last few years, several tests of the validity of the model have been performed (see, for instance, Wegener, 1982; Wegener and Vannahme, 1981). Only one of them will be reported here. In that test gross migration flows predicted by the model were compared with observed migrations recorded by the municipal population registers for the years between 1970 and 1980. Table 4 summarizes the results.

Period	All migration flows			Migration flows < 1,000		
	n	r^2	MAPE[a]	n	r^2	MAPE[a]
1970-1971	961	0.9810	20.7	856	0.4853	71.2
1972-1973	961	0.9708	22.8	850	0.4927	71.2
1974-1975	961	0.9736	25.9	853	0.4198	78.9
1976-1977	961	0.9711	26.9	853	0.2684	89.5
1978-1979	961	0.9572	34.8	855	0.2622	86.1

[a] mean absolute percentage error

Table 4. Model validation results: Goodness-of-fit of predicted migration flows, 1970-1980.

Again the r-square statistic suggests a very good correspondence between observed and predicted flows, even almost a decade away from the base year for which the model was calibrated. Note that no data other than those of the base year and the first simulation period (1970-1972) were used for the calibration. Given the large number of model parameters, the chance of the model's going astray over a decade is far greater than that of its keeping near the actual course of events. In the light of this, the model seems to well reproduce the general pattern of household mobility in the region during the decade. However, the MAPE statistic indicates that the prediction errors for individual flows still are substantial. This is even more obvious if only small flows are inspected. Here the performance of the model is far from being satisfactory and gets worse with time. It remains to be investigated if this failure to correctly predict small migration flows is related to the insufficient resolution of the sampling and aggregation process in the Monte Carlo simulation and hence could be overcome at the expense of more computer time.

CONCLUSIONS

In this chapter, an approach to modeling urban housing markets has been presented which differs in several respects from other contemporary housing market models.

First, it does not subscribe to the hypothesis that during the market clearing process, through price and quantity adjustment, supply and demand arrive at equilibrium. Instead it is built on a set of hypotheses about delayed price and quantity adjustment derived from characteristics of highly regulated European housing markets.

Second, it takes account of the low level of information and high degree of uncertainty characterizing decision situations on the housing market by explicitly modeling the search behavior of households and landlords involving adaptation and learning behavior and satisficing as a decision rule.

Third, it solves the market clearing problem without separating the decision to move from the housing search by using micro simulation with endogenous sampling and re-aggregation.

These features make the model highly realistic and extremely adaptable to new hypotheses and policies. The price to be paid for this realism and adaptability is that it is difficult to calibrate.

It has been attempted to show that the estimation difficulties encountered with the model can be partly overcome by (a) focusing on the constraints restricting the choice situations of model actors such as households or housing investors, and (b) standardizing the parameters of the choice functions in order to make them amenable to discussion and judgment.
It was demonstrated by way of an example that by combining parameters based on intuitive judgment with a few statistically estimated parameters, levels of goodness-of-fit can be achieved which are not inferior to those derived from models entirely estimated statistically from survey data.

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